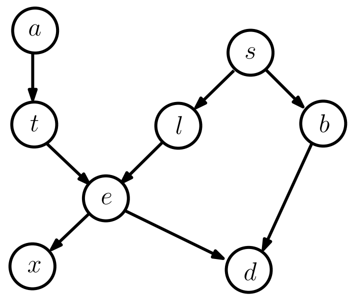
Exercises for Graphical Models in Computer Vision - No. 2

Due Date: 2nd May

Hand in before the exercise (in person or via mail to `joel.janai@tue.mpg.de`).

1. Chest Clinic Network (2 + 3 + 5 Points)



The chest clinic network above concerns the diagnosis of lung disease (tuberculosis,lung cancer, or both, or neiter). In this model a visit to asia is assumed to increase the probability of tuberculosis. We have the following binary variables.

x	positive X-ray
d	Dyspnea (shortness of breath)
e	Either Tuberculosis or Lung Cancer
t	Tuberculosis
l	Lung cancer
b	Bronchitis
a	Visited Asia
s	Smoker

1. Write down the factorization of the distribution implied by the graph.
2. Are the following independence statements implied by the graph? (And how do you conclude this?)
   (a) tuberculosis⫫smoking|shortness of breath
   (b) lung cancer⫫bronchitis|smoking
   (c) visit to Asia⫫smoking|lung cancer
   (d) visit to Asia⫫smoking|lung cancer,shortness of breath
3. Calculate by hand the values for $p(d)$. The Conditional Probability Table (CPT) is:

$$
\begin{aligned}
p(a=1) &= 0.01, & p(s=1) &= 0.5 \\
p(t=1 \mid a=1) &= 0.05, & p(t=1 \mid a=0) &= 0.01 \\
p(l=1 \mid s=1) &= 0.1, & p(l=1 \mid s=0) &= 0.01 \\
p(b=1 \mid s=1) &= 0.6, & p(b=1 \mid s=0) &= 0.3 \\
p(x=1 \mid e=1) &= 0.98, & p(x=1 \mid e=0) &= 0.05 \\
p(d=1 \mid e=1, b=1) &= 0.9, & p(d=1 \mid e=1, b=0) &= 0.7 \\
p(d=1 \mid e=0, b=1) &= 0.8, & p(d=1 \mid e=0, b=0) &= 0.1
\end{aligned}
$$

   and

$$
p(e=1 \mid t,l) = \begin{cases} 0 & t=0 \wedge l=0 \\ 1 & \text{otherwise} \end{cases}
$$

   Important: Think before you start calculating; what is a good sequence of calculations?

2. The Three Door Problem

This is an additional question which does not count, but is interesting to look at from a Bayesian perspective!

The following famous problem is known as the three door problem. In a game show a candidate is offered to choose from three doors. Behind one door there is a prize, behind the other two doors there is a Zonk. The candidate receives whatever is behind the door of his choice and is more interested in the prize than the Zonk.

After the candidates chooses a door and informs the game-show host about it, the host opens one from the other two doors **not** containing the prize. Now the candidate is offered to change his choice, he can again choose freely from the two remaining doors. Should he stick to his first choice or change to the other remaining door?

You are the candidate, and of course you are a Bayesian (at least for this exercise). Explain why you should switch using the notion of random variables and the terms prior, posterior and conditional probability. (Hint: Use three RVs, one for your choice, one for the host's choice and one for the prize). What are the conditional probabilites, your prior, etc?